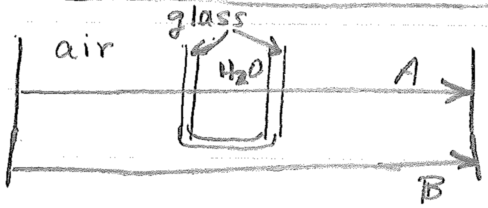
Physics 302 Photonics

HW-6, Chapter 7 — SOLUTIONS

(7.6)



Optical path length of A $\equiv$ $OPL_A = \sum_i n_i d_i$

$$OPL_A = 1(100\,cm - 11\,cm) + 1.52(1.0\,cm) + 1.33(10\,cm)$$

(a) $\boxed{OPL_A = 103.82\ cm}$

(b) $\boxed{OPL_B = 1(100\,cm) = 100\,cm}$

(c) phase difference $= \delta = k\Delta x = \frac{2\pi}{\lambda_0}(OPL_A - OPL_B)$

$$\delta = \frac{2\pi}{500\,nm}(3.82\,cm) \times 10^9 \frac{nm}{m} \times \frac{1\,m}{10^2\,cm}$$

$$\boxed{\delta = 4.8 \times 10^5 \text{ radians}}$$

(7.27) From Eq. 7.38, $v_g = v + k\frac{dv}{dk}$

But $\frac{dv}{dk} = \frac{d\lambda}{dk}\frac{dv}{d\lambda}$. Also $\lambda = \frac{2\pi}{k}$ and $v = \frac{c}{n}$

$$\frac{dv}{dk} = \frac{d}{dk}\left(\frac{2\pi}{k}\right)\frac{d}{d\lambda}\left(\frac{c}{n}\right) = \frac{2\pi c}{k^2 n^2}\frac{dn}{d\lambda}$$

$$\Rightarrow v_g = \frac{c}{n} + k\left(\frac{2\pi c}{k^2 n^2}\frac{dn}{d\lambda}\right) = \frac{c}{n} + \frac{2\pi c}{k\, n^2}\frac{dn}{d\lambda}$$

or $$\boxed{v_g = \frac{c}{n} + \frac{\lambda c}{n^2}\frac{dn}{d\lambda}}$$

(7.29) Given: $\omega^2 = \omega_p^2 + c^2k^2$ for an ionized plasma with ω_p = plasma freq.

(a) The group velocity is $v_g = \left(\frac{d\omega}{dk}\right)_{\bar{\omega}}$

or $v_g = \frac{d}{dk}(\omega_p^2 + c^2k^2)^{1/2}$

$$v_g = \frac{c^2k}{(\omega_p^2 + c^2k^2)^{1/2}} = \frac{c^2k}{\omega}$$

(b) The phase velocity is $v = \omega/k$, so

(c) $$v\,v_g = \left(\frac{\omega}{k}\right)\left(\frac{c^2k}{\omega}\right) = c^2$$

(7.42) Coherence length $\Delta l_c = c\Delta t_c = \frac{c}{\Delta \nu_c}$

Given: $\frac{\Delta \nu_c}{\bar{\nu}} = \frac{2}{10^{10}} = 2\times10^{-10}$

$$\Rightarrow \Delta \nu_c = 2\times10^{-10}\,\bar{\nu}$$

So $\Delta l_c = \frac{c}{2\times10^{-10}\,\bar{\nu}} = \frac{\bar{\lambda}}{2\times10^{-10}} = \frac{632.8\text{ nm}}{2\times10^{-10}}$

$$\boxed{\Delta l_c = 3164\text{ m}}$$

(7.46) $\Delta\lambda = 1.2\text{ nm}$ and $\bar{\lambda} = 500\text{ nm}$

$$\Delta\lambda = \Delta\left(\frac{c}{\nu}\right) = \frac{c}{\nu^2}\Delta\nu$$

$$\Rightarrow \Delta\nu = \frac{\nu^2}{c}\Delta\lambda = \frac{c\Delta\lambda}{\lambda^2}$$

$$= \frac{(3.0\times10^{8}\frac{m}{s})(1.2\text{ nm})}{(50\text{nm})^2}$$

(a) $\boxed{\Delta\nu = 1.4\times10^{12}\text{ Hz}}$

(b) $\boxed{\Delta l_c = \frac{c}{\Delta\nu} = 2.1\times10^{-4}\text{ m} = 2.1\times10^{5}\text{ nm}}$